CrackSAT.net

SAT II Math Level 2 Study Guide

SAT MATH LEVEL 2 SPARKNOTES TEST PREP

Orientation

1.0 Introduction to the SAT II

2.0 Content and Format of the SAT II Math IIC

3.0 Strategies for SAT II Math IIC

Math IIC Review

4.0 Math IIC Fundamentals

5.0 Algebra

6.0 Plane Geometry

7.0 Solid Geometry

8.0 Coordinate Geometry

9.0 Trigonometry

10.0 Functions

11.0 Statistics

12.0 Miscellaneous Math

Practice Tests

13.0 Practice Tests Are Your Best Friends

Math IIC Scoring

Scoring on the SAT II Math IIC is very similar to the scoring for all other SAT II tests. For every right answer, you earn 1 point. For every wrong answer, you lose $^1/_4$ of a point. For every question you leave blank, you earn 0 points. Add these points up, and you get your raw score. ETS then converts your raw score to a scaled score according to a special curve. We have included a generalized version of that curve in the table below. Note that the curve changes slightly for each edition of the test, so the table shown will be close to, but not exactly the same as, the table used by the ETS for the particular test you take. You should use this chart to convert your raw scores on practice tests into a scaled score.

Scaled Score	Average Raw Score	Scaled Score	Average Raw Score
800	50	570	18
800	49	560	17
800	48	550	16
800	47	540	15
800	46	530	14
800	45	520	13
800	44	510	12

CrackSAT.net

Useful Links:

SAT Online Practice Tests: http://www.cracksat.net/tests/

SAT Subjects Tests: http://www.cracksat.net/sat2/

SAT Downloads: http://www.cracksat.net/sat-downloads/

For more SAT information, please visit http://www.cracksat.net

SAT Downloads:

SAT real tests download:

http://www.cracksat.net/sat-downloads/sat-real-tests.html

SAT official guide tests download:

http://www.cracksat.net/sat-downloads/sat-official-guide-tests.html

SAT online course tests download:

http://www.cracksat.net/sat-downloads/sat-online-course-tests.html

SAT subject tests download:

http://www.cracksat.net/sat-downloads/sat-subject-tests.html

PSAT real tests download:

http://www.cracksat.net/psat/download/

1000+ College Admission Essay Samples:

http://www.cracksat.net/college-admission/essays/

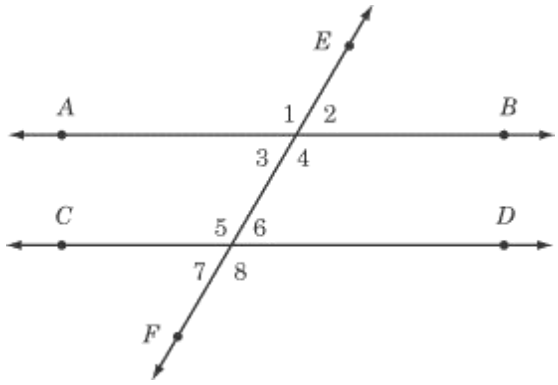
CrackSAT.net

800	43	500	11
790	42	490	10
780	41	480	9
770	40	470	8
760	39	450	7
750	38	440	6
740	37	430	5
730	36	420	4
720	35	410	3
710	34	400	2
700	33	390	1
690	32	380	0
680	31	370	−1
680	30	360	−2
670	29	350	−3
660	28	340	−4
650	27	330	−5
640	26	320	−6
630	25	310	−7
630	24	300	−8
620	23	300	−9
610	22	290	−10
600	21	290	−11

590	20	280	−12
580	19	280	−13

Parallel Lines

Lines that don't intersect are called parallel lines. The intersection of one line with two parallel lines creates many interesting angle relationships. This situation is often referred to as "parallel lines cut by a transversal," where the transversal is the nonparallel line. As you can see in the diagram below of parallel lines *AB* and *CD* and transversal *EF*, two parallel lines cut by a transversal will form 8 angles.



Among the eight angles formed, three special angle relationships exist:

1. **Alternate exterior angles are pairs of congruent angles on opposite sides of the transversal, outside of the space between the parallel lines. In the figure above, there are two pairs of alternate exterior angles: $\angle 1$ and $\angle 8$, and $\angle 2$and $\angle 7$.**
2. **Alternate interior angles are pairs of congruent angles on opposite sides of the transversal in the region between the parallel lines. In the figure above, there are two pairs of alternate interior angles: $\angle 3$ and $\angle 6$, and $\angle 4$ and $\angle 5$.**
3. **Corresponding angles are congruent angles on the same side of the transversal. Of two corresponding angles, one will always be between the parallel lines, while the other will be outside the parallel lines. In the figure above, there are four pairs of corresponding angles: $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, and $\angle 4$ and $\angle 8$.**

In addition to these special relationships between angles formed by two parallel lines cut by a transversal, all adjacent angles are supplementary.

Polygons

Polygons are enclosed geometric shapes that cannot have fewer than three sides. As this definition suggests, triangles are actually a type of polygon, but they are so important on the Math IIC that they merit their own section. Polygons are named according to the number of sides they have, as you can see in the following chart:

CrackSAT.net

Number of Sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	n-gon

All polygons, no matter the number of sides they possess, share certain characteristics:

- The sum of the interior angles of a polygon with n sides is $(n - 2)180^o$. So, for example, the sum of the interior angles of an octagon is $(8 - 2)180^o = 6(180^o) = 1080^o$.
- The sum of the exterior angles of any polygon is 360^o.
- The perimeter of a polygon is the sum of the lengths of its sides.

Prism: 柱体 **cylinder:** 圆柱体 **cone:** 圆锥体 **pyramid:** 锥体

Surface Area of a Cone

The surface area of a cone consists of the lateral surface area and the area of the base. The base is a circle and therefore has an area of πr^2. The lateral surface is the cone "unrolled." Depending on the shape of the cone, it can be the shape of a triangle with a curved base, a half circle, or a "Pacman" shape. The area of the lateral surface is a length that is related to the circumference of the circle times the lateral height, l. This is the formula:

$$\text{Lateral Surface Area of a Cone} = \pi rl$$

where r is the radius and l is the lateral height. An alternate formula for lateral surface area is:

$$\text{Lateral Surface Area of a Cone} = \pi \frac{1}{2} cl$$

where c is the circumference and l is still the lateral height. The two formulas for surface area are the same, since the circumference of a circle is equal to $2\pi r$. But knowing one or the other can save time.

The *total* surface area of a cone is the sum of the base area and lateral surface area:

$$\text{Total Surface Area of a Cone} = \pi r^2 + \pi rl$$

Surface Area of a Pyramid

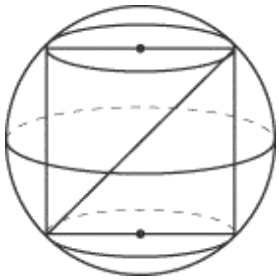
The surface area of a pyramid is rarely tested on the Math IIC test. If you do come across a question that covers the topic, you can calculate the area of each face individually using techniques from plane geometry, since the base of the pyramid is a square and the sides are triangles.

The Rules of Inscribed Solids

Math IIC questions involving inscribed solids are much easier to solve when you know how the lines of different solids relate to one another. For instance, the previous example showed that when a cube is inscribed in a cylinder, the diagonal of a face of the cube is equal to the diameter of the cylinder. The better you know the rules of inscribed solids, the better you'll do on these questions. Here are the rules of inscribed solids that most commonly appear on the Math IIC.

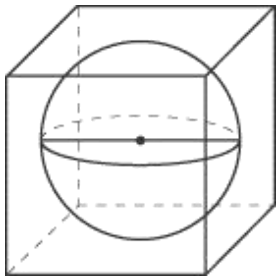
Cylinder Inscribed in a Sphere

The diameter of the sphere is equal to the diagonal of the cylinder's height and diameter.



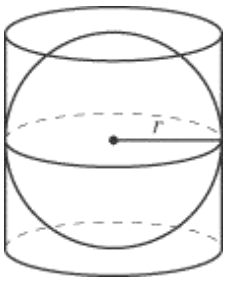
Sphere Inscribed in a Cube

The diameter of the sphere is equal to the length of cube's edge.



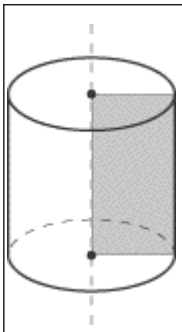
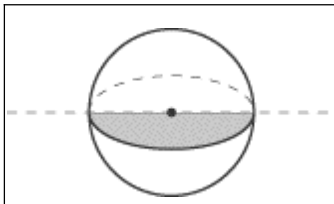
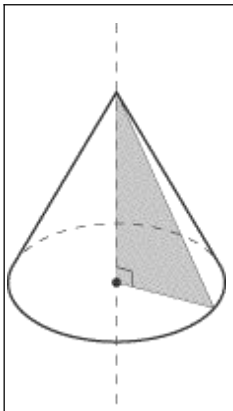
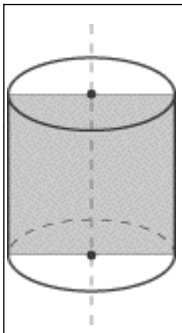
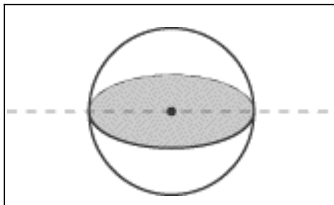
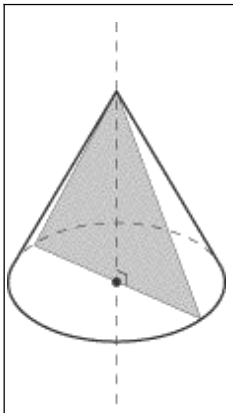
Sphere Inscribed in a Cylinder

The cylinder and the sphere have the same radius.



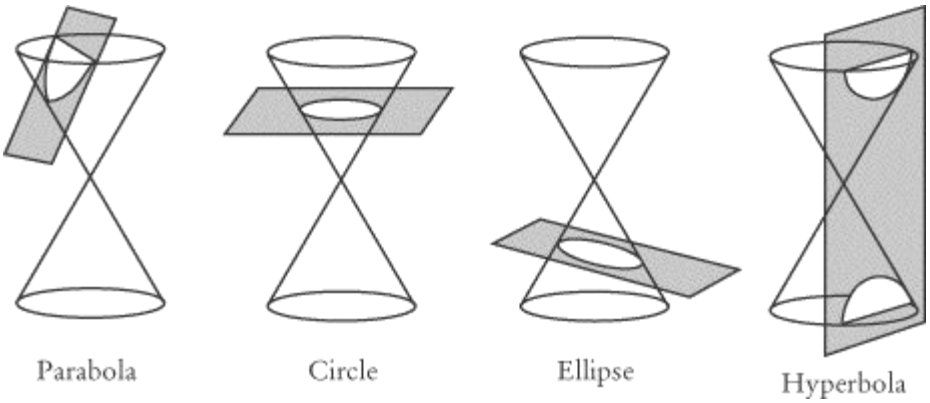
Common Rotations

You don't need to learn any new techniques or formulas for problems dealing with rotating figures. You just have to be able visualize the described rotation and be aware of which parts of the polygons become which parts of the geometric solid. Below is a summary of which polygons, when rotated a specific way, produce which solids.

		
A rectangle rotated about its edge produces a cylinder.	A semicircle rotated about its diameter produces a sphere.	A right triangle rotated about one of its legs produces a cone.
		
A rectangle rotated about a central axis (which must contain the midpoint of both of the sides that it intersects) produces a cylinder.	A circle rotated about its diameter produces a sphere.	An isosceles triangle rotated about its axis of symmetry (the altitude from the vertex of the non-congruent angle) produces a cone.

Other Important Graphs and Equations

In addition to the graphs and equations of lines, the Math IIC will test your understanding of the graphs and equations of parabolas, circles, ellipses, and hyperbolas.



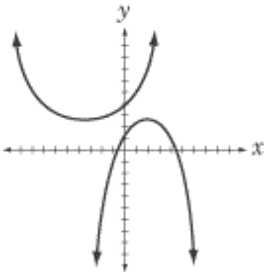
CrackSAT.net

Questions on these topics either will ask you to match up the correct graph with the correct equation or give you an equation and ask you to figure out certain characteristics of the graph.

Most of the questions about conic sections are straightforward. If you know the information in the sections below, you'll be able to breeze through them.

Parabolas

A parabola is a U-shaped curve that can open either upward or downward.



A parabola is the graph of a quadratic function, which, you may recall, is $ax^2 + bx + c$. The equation of a parabola can take on two different forms—the standard form and the general form. Each can help you determine different information about the nature of the parabola.

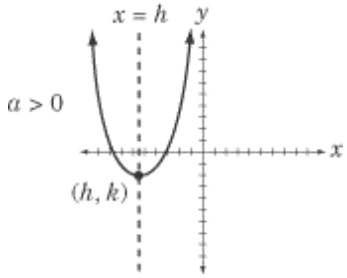
Standard Form of the Equation of a Parabola

The standard form of the equation of a parabola is perhaps the most useful and will be the one most used on the Math IIC test:

$$y = a(x - h)^2 + k$$

where a, h, and k are constants. From this formula, you can learn a few pieces of information:

1. The vertex of the parabola is (h, k).
2. The axis of symmetry of the parabola is the line $x = h$.
3. The parabola opens upward if $a > 0$ and downward if $a < 0$.



For example, if you were given the parabola equation $y = -3(x - 5)^2 + 8$, you first need to pick out the values of the constants a, h, and k. Then you can derive information about the parabola. For this example, $a = -3$, $h = 5$, and $k = 8$. So the vertex is (5, 8), the axis of symmetry is the line $x = 5$, and since $-3 < 0$, the parabola opens downward.

General Form of the Equation of a Parabola

The general form of the equation of a parabola is:

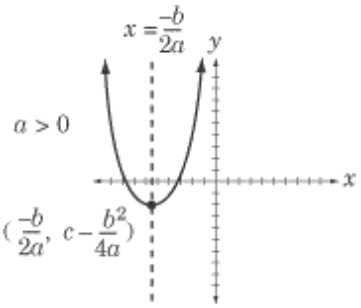
$$y = ax^2 + bx + c$$

where a, b, and c are constants. If a question presents you with a parabola equation in this form, you can find out the following information about the parabola:

1. The vertex of the parabola is $(-b/2a, c - b^2/4a)$.
2. The axis of symmetry of the parabola is the line $x = -b/2a$.
3. The parabola opens upward if $a > 0$ and downward if $a < 0$.

CrackSAT.net

4. The y-intercept is the point $(0, c)$.



Circles

A circle is the collection of points equidistant from a given point, called the center of the circle. For the Math IIC test, there is only one equation you have to know for a circle. This equation is called the standard form:

$$(x - h)^2 + (y - k)^2 = r^2$$

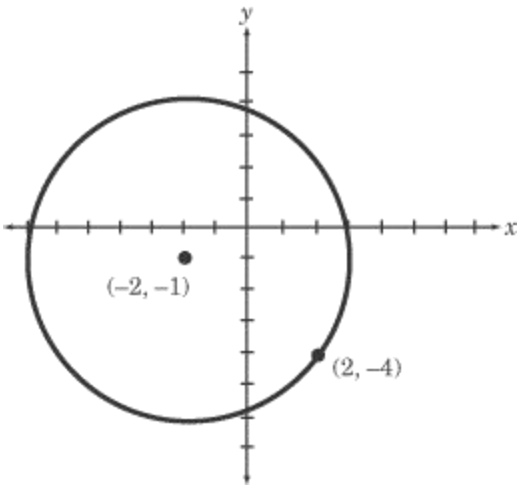
where (h, k) is the center of the circle and r is the radius. When the circle is centered at the origin, so that $h = k = 0$, then the equation simplifies to:

$$x^2 + y^2 = r^2$$

That's it. That's all you need to know about a circle in coordinate geometry. Once you know and understand this equation, you should be able to sketch a circle in its proper place on the coordinate system if given its equation. You will also be asked to figure out the equation of a circle given a picture of its graph.

To see if you know what you need to know, try to answer the following practice problem:

What is the equation of the circle pictured below?



The center is given in the image: (–2 ,–1). All you need to finish the formula is the radius. We do this by finding the distance from the center and the point (2, –4) pictured on the circle:

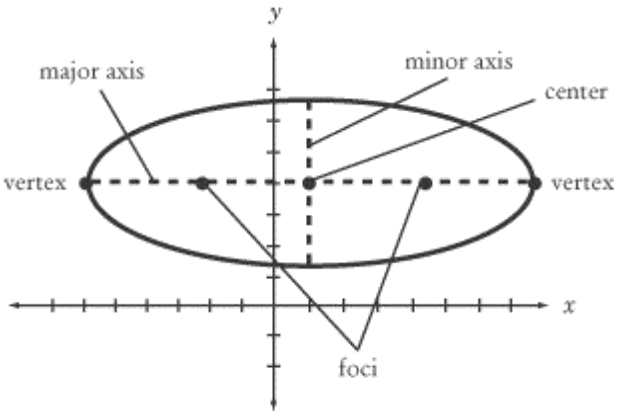
$$r = \sqrt{(2 - (-2))^2 + (-4 - (-1))^2} = \sqrt{4^2 + 3^2} = 5$$

The radius of the circle is 5, so the equation of the circle can be written as $(x + 2)^2 + (y + 1)^2 = 25$.

Ellipses

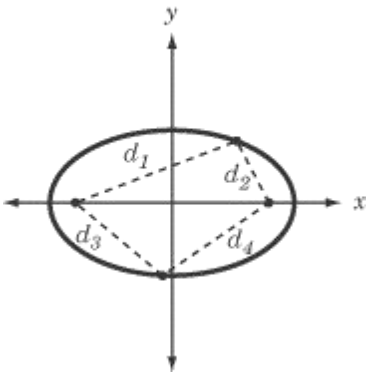
CrackSAT.net

An ellipse is a figure shaped like an oval. It looks like a circle somebody sat on, but it's a good deal more complicated than a circle, as you can see from the diagram below.



An ellipse is formed by a set of points at a constant summed distance from two fixed points called the foci. The line segment containing the foci of an ellipse with both endpoints on the ellipse is called the major axis. The endpoints of the major axis are called the vertices. The line segment perpendicular to the major axis with both endpoints on the ellipse is the minor axis. The point halfway between the foci is the center of the ellipse. When you see an ellipse, you should be able to identify where each of these components would be.

The two foci are crucial to the definition of an ellipse. The sum of the distances from both foci to any point on the ellipse is constant. For every point on the ellipse, the cumulative distance from the two foci to that point will be constant. In the image below, for example, $d_1 + d_2$ is equal to $d_3 + d_4$.



The standard form of the equation of an ellipse is:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

where a, b, h, and k are constants. With respect to this formula, remember that:

1. The center of the ellipse is (h, k).
2. The length of the horizontal axis is $2a$.
3. The length of the vertical axis is $2b$.
4. If $a > b$, the major axis is horizontal and the minor axis is vertical; if $b > a$, the major axis is vertical and the minor axis is horizontal.

When an ellipse is centered at the origin so that $h = k = 0$, the standard form of the equation of an ellipse becomes:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

On the test, you might see a question like this:

What are the coordinates of the center and vertices of an ellipse given by the following equation?

CrackSAT.net

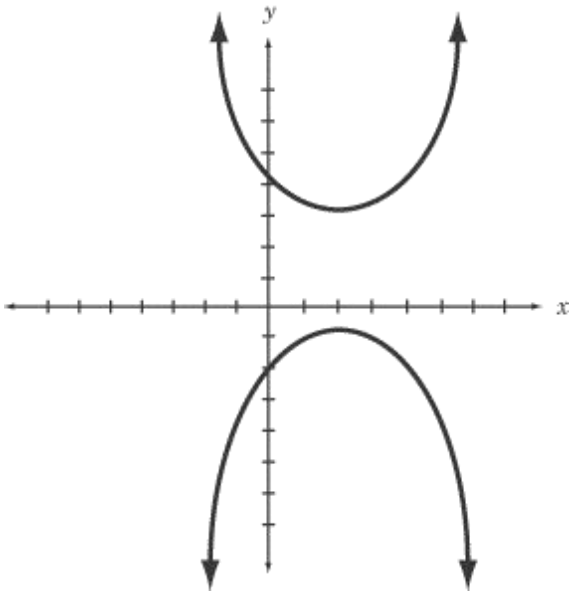
$$\frac{(x-2)^2}{16} - \frac{(y+5)^2}{36} = 1$$

First, find the center of the ellipse. By comparing this equation to the standard form, you see that $(h, k) = (2, -5)$. Since the vertices are the endpoints of the major axis, your next step should be to find the orientation and length of that axis. In this ellipse, $b > a$, so the major axis is vertical and is $2b = 2\sqrt{36} = 12$ units long. The coordinates of the vertices are therefore $(2, -5 \pm 6)$, which works out to $(2, -11)$ and $(2, 1)$.

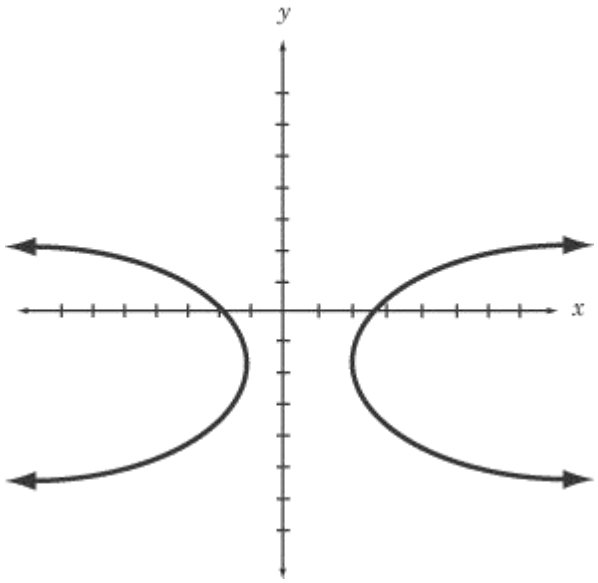
Hyperbolas

Though hyperbolas appear infrequently on the Math IIC, you should still review them.

A hyperbola is shaped like two parabolas facing away from each other:



The two parts of a hyperbola can open upward and downward, like they do in the previous graph, or they can open to the sides, like the hyperbola below:



The standard form of the equation of a hyperbola that opens to the sides is:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

where a, b, h, and k are constants. The standard form of the equation of a hyperbola that opens upward and downward is the same as the side form except that the $(x - h)$ and $(y - k)$ terms are interchanged:

CrackSAT.net

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

The center of a hyperbola is (h, k), and the axis of symmetry is the line $x = h$ for vertical hyperbolas or $y = k$ for horizontal hyperbolas.

Similar to the equations for a circle and ellipse, the equation of a hyperbola becomes simpler for a hyperbola centered at the origin:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

If you see a question about a hyperbola, it will most likely concern the center of the hyperbola, which can be readily found using the equation of the hyperbola.

Function	Domain	Range
$y = \sin(x)$	$-\infty < x < \infty$	$-1 \le y \le 1$
$y = \cos(x)$	$-\infty < x < \infty$	$-1 \le y \le 1$
$y = \tan(x)$	$-\infty < x < \infty, x \ne n\pi + \pi/2;\ n$ is an integer	$-\infty < y < \infty$
$y = \csc(x)$	$-\infty < x < \infty, x \ne n\pi;\ n$ is an integer	$y \le -1$ or $y \ge 1$
$y = \sec(x)$	$-\infty < x < \infty, x \ne n\pi + \pi/2;\ n$ is an integer	$y \le -1$ or $y \ge 1$
$y = \cot(x)$	$-\infty < x < \infty, x \ne n\pi;\ n$ is an integer	$-\infty < y < \infty$

- For $y = \sin x$ and $y = \cos x$, the period is 2π radians. This means that every 360°, the values of sine and cosine repeat themselves. For example, trigonometric functions of 0 and 2π radians produce the same values.
- For $y = \tan x$, the period is π radians. Thus, the tangents of 0° and 180° are equal.

If a trigonometric function contains a coefficient in front of x, its period changes. In general, the period of $y = f(bx)$ is the normal period of f divided by b. For example, the period of $y = \sin {}^1/_4 x = 2\pi$ radians $\div {}^1/_4 = 8\pi$ radians.

Finding Whether the Inverse of a Function Is a Function

Take a look at this question:

Is the inverse of $f(x) = x^2$ a function?

To answer a question like this, you must, of course, first find the inverse. In this case, begin by writing $y = x^2$. Next, switch the places of x and y: $x = y^2$. Solve for y: $y = \sqrt{x}$. Now you need to analyze the inverse of the function and decide whether for every x, there is only one y. If only one y is associated with each x, you've got a function. Otherwise, you don't. For functions, the square root of a quantity equals the positive root only; in fact, all even-numbered roots in functions have only positive values. In this case, every x value that falls within the domain $x \ge 0$ turns out one value for y, so $f^{-1}(x)$ is a function.

Here's another sample question:

What is the inverse of $f(x) = 2|x - 1|$, and is it a function?

Again, replace x with y and solve for y:

CrackSAT.net

$$x = 2|y - 1|$$
$$\frac{1}{2}x = |y - 1|$$

Now, since you're dealing with an absolute value, split the equations:

$$y - 1 = \frac{1}{2}x \text{ or } 1 - y = \frac{1}{2}x$$

Therefore,

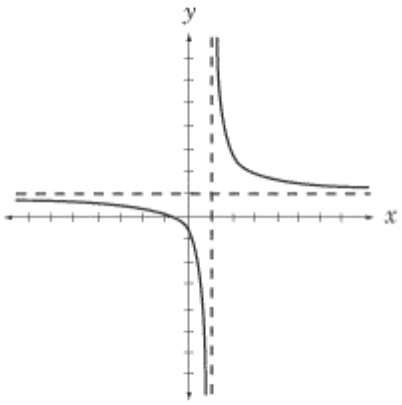
$$y = \frac{1}{2}x + 1 \text{ or } y = -\frac{1}{2}x + 1$$

The inverse of $f(x)$ is this set of two equations. As you can see, for every value of x, except 0, the inverse of the function assigns two values of y. Consequently, $f^{-1}(x)$ is not a function.

Asymptotes and Holes

There are two types of abnormalities that can further limit the domain and range of a function: asymptotes and holes. Being able to identify these abnormalities will help you to match up the domain and range of a graph to its function.

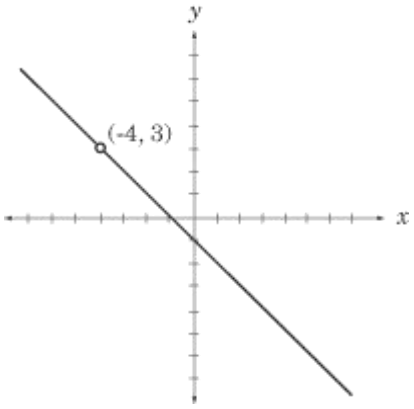
An asymptote is a line that a graph approaches but never intersects. In graphs, asymptotes are represented as dotted lines. You'll probably only see vertical and horizontal asymptotes on the Math IIC, though they can exist at other slopes as well. A function is undefined at the x value of a vertical asymptote, thus restricting the domain of the function graphed. A function's range does not include the y value of a horizontal asymptote, since the whole point of an asymptote is that the function never actually takes on that value.



In this graph, there is a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 1$. Because of these asymptotes, the domain of the graphed function is the set of real numbers except 1 ($x \neq 1$), and the range of the function graphed is also the set of real numbers except 1 ($f(x) \neq 1$).

A hole is an isolated point at which a function is undefined. You'll recognize it in a graph as an open circle at the point where the hole occurs. Find it in the following figure:

CrackSAT.net

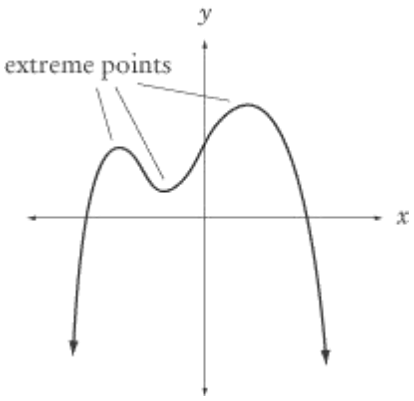


The hole in the graph above is the point (–4, 3). This means that the domain of the function is the set of real numbers except 4 ($x \neq -4$), and the range is the set of real numbers except 3 ($f(x) \neq 3$).

Degree

The degree of a polynomial function is the highest exponent to which the dependent variable is raised. For example, $f(x) = 4x^5 - x^2 + 5$ is a fifth-degree polynomial, because its highest exponent is 5.

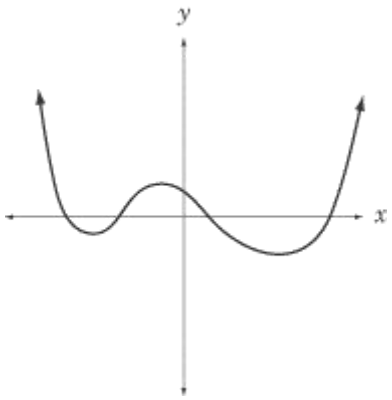
A function's degree can give you a good idea of its shape. The graph produced by an n-degree function can have as many as $n - 1$ "bumps" or "turns." These "bumps" or "turns" are technically called "extreme points."



Once you know the degree of a function, you also know the greatest number of extreme points a function can have. A fourth-degree function can have at most three extreme points; a tenth-degree function can have at most nine extreme points.

If you are given the graph of a function, you can simply count the number of extreme points. Once you've counted the extreme points, you can figure out the smallest degree that the function can be. For example, if a graph has five extreme points, the function that defines the graph must have at least degree six. If the function has two extreme points, you know that it must be at least third degree. The Math IIC will ask you questions about degrees and graphs that may look like this:

CrackSAT.net



If the graph above represents a portion of the function $g(x)$, then which of the following could be $g(x)$?

(A) a

(B) $ax + b$

(C) $ax^2 + bx + c$

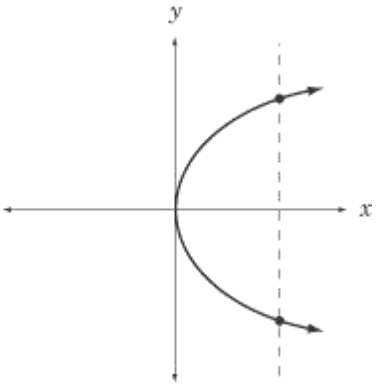
(D) $ax^3 + bx^2 + cx + d$

(E) $ax^4 + bx^3 + cx^2 + dx + e$

To answer this question, you need to use the graph to learn something about the degree of the function. Since the graph has three extreme points, you know the function must be at least of the fourth degree. The only function that fits that description is E. Note that the answer could have been any function of degree four or higher; the Math IIC test will never present you with more than one right answer, but you should know that even if answer choice E had read $ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h$ it still would have been the right answer.

Symmetry Across the *x*-Axis

No function can have symmetry across the x-axis, but the Math IIC will occasionally include a graph that is symmetrical across the x-axis to fool you. A quick check with the vertical line test proves that the equations that produce such lines are not functions:



Statistical Analysis

On the Math IIC you will occasionally be presented with a data set—a collection of measurements or quantities. For example, the set of test scores for the 20 students in Ms. McCarthy's math class is a data set.

71, 83, 57, 66, 95, 96, 68, 71, 84, 85, 87, 90, 88, 90, 84, 90, 90, 93, 97, 99

From a given a data set, you should be able to derive the four following values:

CrackSAT.net

1. Arithmetic mean
2. Median
3. Mode
4. Range

Arithmetic Mean

The arithmetic mean is the value of the sum of the elements contained in a data set divided by the number of elements found in the set.

$$\text{Arithmetic Mean} = \frac{\text{the sum of the elements of a set}}{\text{the number of elements in the set}}$$

On the Math IIC and in many high school math classes, the arithmetic mean is often called an "average" or is simply referred to as the mean.

Let's take another look at the test scores of Ms. McCarthy's math class. We've sorted the scores in her class in order from lowest to highest:

57, 66, 68, 71, 71, 83, 84, 84, 85, 87, 88, 90, 90, 90, 90, 93, 95, 96, 97, 99

To find the arithmetic mean of this data set, we must sum the scores, and then divide by 20, since that is the number of scores in the set. The mean of the math test scores in Ms. McCarthy's class is:

$$\text{mean} = \frac{57 + 66 + 68 + \cdots + 96 + 97 + 99}{20}$$
$$\text{mean} = \frac{1600}{20}$$
$$\text{mean} = 80$$

While some Math IIC questions might cover arithmetic mean in the straightforward manner shown in this example, it is more likely the test will cover mean in a more complicated way.

The Math IIC might give you $n - 1$ numbers of an n number set and the average of that set and ask you to find the last number. For example:

> **If the average of four numbers is 22 and three of the numbers are 7, 11, and 18, then what is the fourth number?**

Remember that the mean of a set of numbers is intimately related to the number of terms in the set and the sum of those terms. In the question above, you know that the average of the four numbers is 22. This means that the four numbers, when added together, must equal 4×22, or 88. Based on the sum of the three terms you are given, you can easily determine the fourth number by subtraction:

$$7 + 11 + 18 + \text{unknown number} = 88$$

Solving for the unknown number is easy: all you have to do is subtract 7, 11, and 18 from 88 to get 52, which is the answer.

The test might also present you with what we call an "adjusted mean" question. For example:

> **The mean age of the 14 members of a ballroom dance class is 34. When a new student enrolled, the mean age increased to 35. How old is th student?**

This question is really just a rephrasing of the previous example. Here you know the original number of students in the class and the original mean of the students' ages, and you are asked to determine the mean after an additional term is introduced. To figure out the age of the new student, you simply need to find the sum of the ages in the adjusted class (with one extra student) and subtract from that the sum of the ages of the original class. To calculate the sum of the ages of the adjusted class:

$$\text{New Mean} = \frac{\text{The new sum of the students' ages}}{\text{The new class enrollment}}$$

CrackSAT.net

$$35 = \frac{\text{The new sum of the students' ages}}{\text{The old class enrollment + 1 new student}}$$

$$35 = \frac{\text{The new sum of the students' ages}}{14 + 1}$$

$$\text{The new sum of the students' ages} = 15 \times 35$$

$$\text{The new sum of the students' ages} = 525$$

By the same calculations, the sum of the students' ages in the original class is $14 \times 34 = 476$. So the new student added an age of $525 - 476 = 49$ years.

Median

The Math IIC might also ask you about the median of a set of numbers. The median is the number whose value is in the middle of the numbers in a particular set.

Take the set 6, 19, 3, 11, 7. Arranging the numbers in order of value results in the list below:

3, 6, 7, 11, 19

Once the numbers are listed in this ordered way, it becomes clear that the middle number in this group is 7, making 7 the median.

In a set with an even number of items it's impossible to isolate a single number as the median, so calculating the median requires an extra step. For our purposes, let's add an extra number to the previous example to produce an even set:

3, 6, 7, 11, 15, 19

When the set contains an even number of elements, the median is found by taking the mean of the two middle numbers. The two middle numbers in this set are 7 and 11, so the median of the set is $^{7+11}/_{2} = 9$.

Mode

The mode is the element of a set that appears most frequently. In the set 10, 11, 13, 11, 20, the mode is 11 since it appears twice and all the other numbers appear just once. In a set where more than one number appears with the same highest frequency, there is more than one mode: the set 2, 2, 3, 4, 4 has modes of 2 and 4. In a set where all of the elements appear an equal number of times, there is no mode.

The good news is that mode questions are easy. The bad news is that mode questions don't appear all that much on the Math IIC.

Range

The range measures the spread of a data set, or the difference between the smallest element and the largest. For the set of test scores in Ms. McCarthy's class:

57, 66, 68, 71, 71, 83, 84, 84, 85, 87, 88, 90, 90, 90, 90, 93, 95, 96, 97, 99

The range is $99 - 57 = 42$.

Logic

Logic questions don't look like math questions at all; they don't contain numbers, formulas, or variables. Instead, logic questions contain a verbal statement and a question that asks you to interpret the validity of the statement or the effect of the given statement on another statement. For example:

The statement "If Jill misses the bus, she will be late" is true. Which other statement must be true?

(A) If Jill does not miss the bus, she will not be late

(B) If Jill is not late, she missed the bus

(C) If a student misses the bus, he or she will be late

(D) Jill is late because she missed the bus

(E) If Jill is not late, she did not miss the bus

Even though they are stated in words, logic questions require mathematical methods for finding the right answers.

A logic statement is written in the form "If p, then q," where p and q are events. "If p, then q" can also be written as $p \rightarrow q$, and it states that if event p occurs, then event q will also occur.

Every "If p, then q" statement has an equivalent statement; this second statement is known as the contrapositive, which is always true. The contrapositive of "If p, then q" is "If not q, then not p." In symbols, the contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$ (here, the symbol ~ means "not"). To formulate the contrapositive of any logic statement, you must change the original statement in two ways.

1. Switch the order of the two parts of the statement. For example, "If p, then q" becomes "If q, then p."
2. Negate each part of the statement. "If q, then p" becomes "If not q, then not p."

When faced with a logic problem on the Math IIC, remember that if a given statement is true, then that statement's contrapositive is also true. Likewise, if a given statement is false, then that statement's contrapositive is also false.
Returning to the example problem, we are told that the given statement is true, so we should look for the contrapositive among the answer choices. E is the contrapositive of the original statement, so we know that it is true. Here's some more practice:

What is the contrapositive of "Every book on the shelf is old"?

You need to first rewrite this statement so that it is in the "If p, then q" form. So the given statement becomes "If a book is on the shelf, then it is old." The contrapositive of the statement is now easy to see: "If a book is not old, then it is not on the shelf."